1. a) i) BASIS: All propositional atoms p are wffs in $\mathcal{L}_{\leftrightarrow}$
ii) IND: If A and B are wffs in $\mathcal{L}_{\leftrightarrow}$, then so is $A \leftrightarrow B$
iii) CLOSURE: Nothing is a wff of $\mathcal{L}_{\leftrightarrow}$ unless it can be generated in finitely many steps from i, ii.

b) BASIS: Each atom p is satisfiable. Simply take a valuation v with $v(p) = 1$ for all atoms p

INDUCTION HYPOTHESIS: Take v s.t. $v(p) = 1$ for ~~all p!~~ Suppose A and B are any wffs in $\mathcal{L}_{\leftrightarrow}$ which are both satisfiable by the v defined above, i.e. $v(A) = 1$ and $v(B) = 1$.

INDUCTIVE STEP: For the v defined above, we have $v(A) = 1$ and $v(B) = 1$, so $v(A \leftrightarrow B) = 1$. Therefore $A \leftrightarrow B$ is satisfiable.

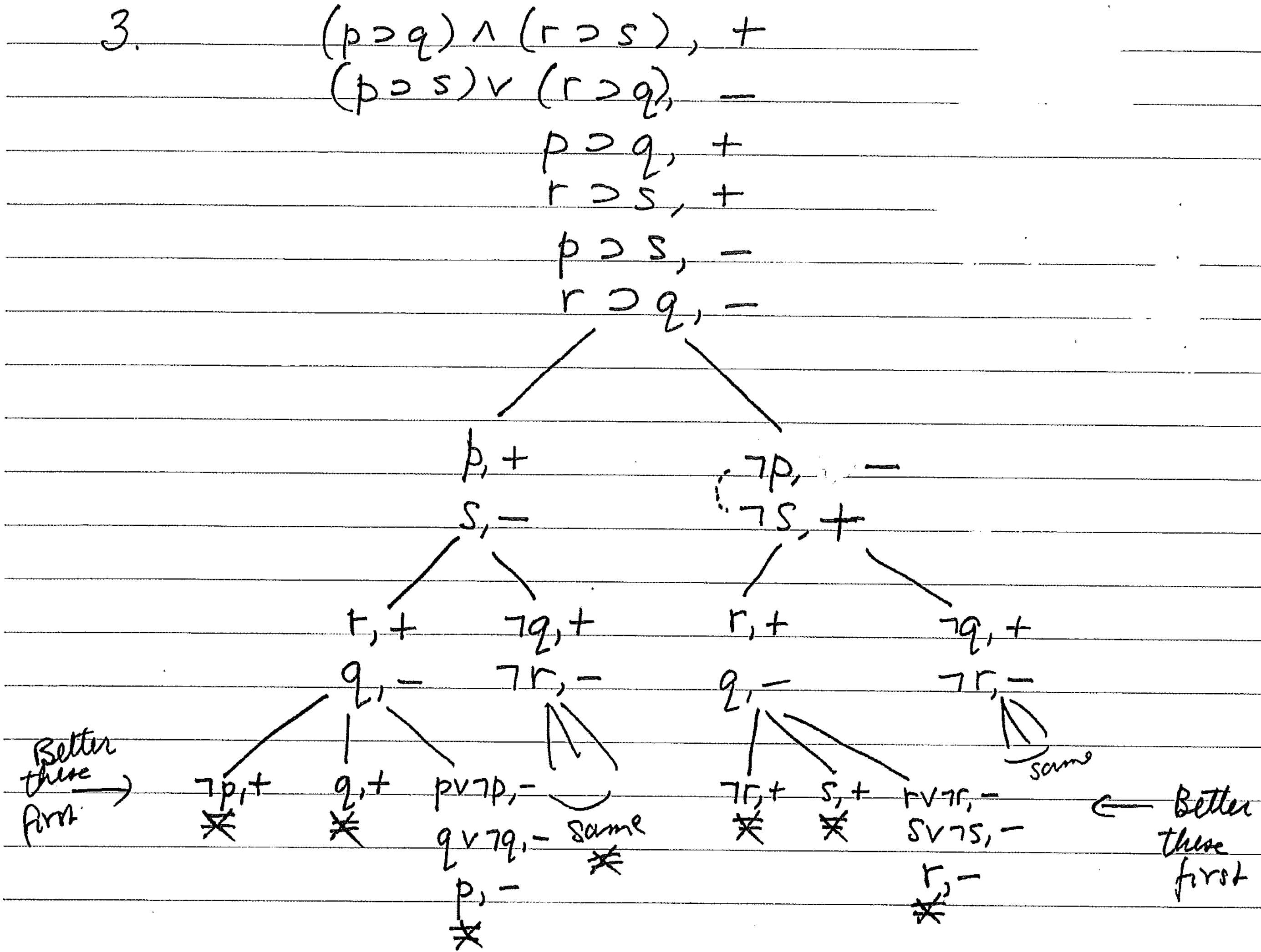
Conclusion: If we define v such that $v(p) = 1$ for all atoms p, then $v(A) = 1$ for all wffs in $\mathcal{L}_{\leftrightarrow}$. Therefore, all wffs in $\mathcal{L}_{\leftrightarrow}$ are satisfiable.

2.

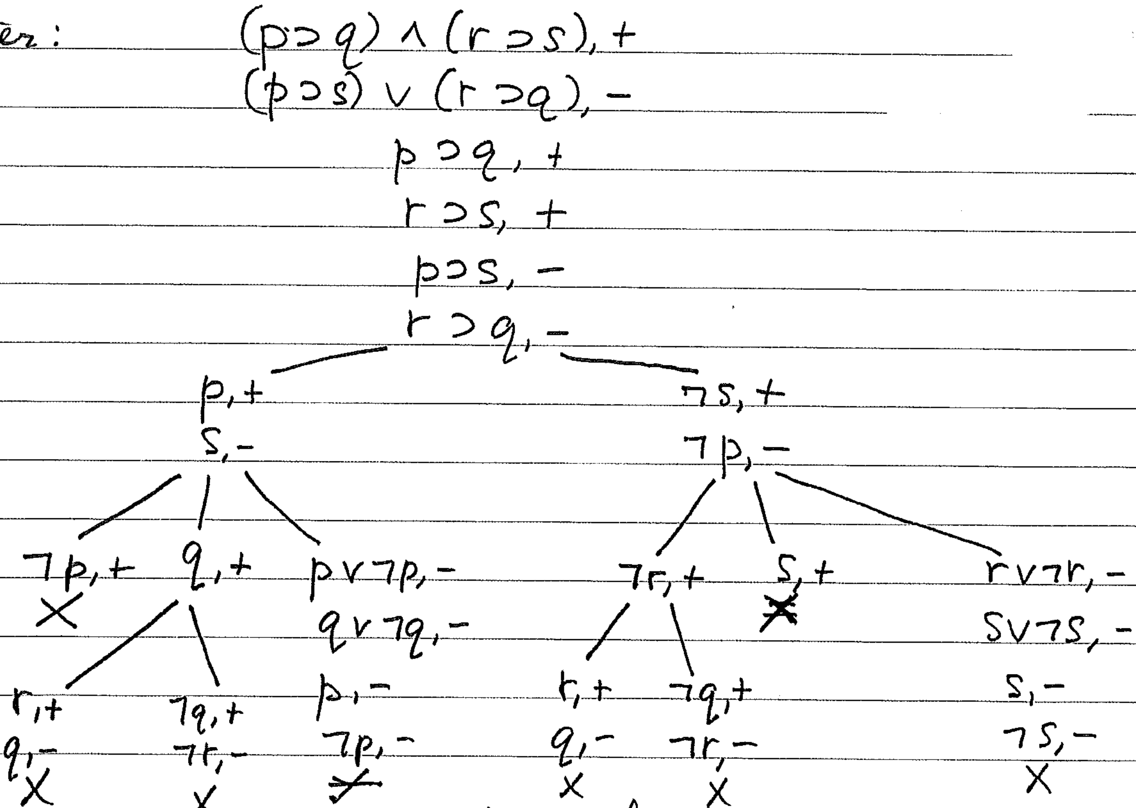
p	q	$(p\wedge q)$	$\wedge$	$\neg p$	$(p\vee q)$	$\supset$	$\neg p$
1	1	1	0	0	1	0	0
1	i	i	0	0	1	0	0
1	0	0	0	0	1	0	0
i	1	i	i	i	1	i	i
i	i	i	i	i	i	i	i
i	0	0	0	i	i	i	i
0	1	0	0	1	1	1	1
0	i	0	0	1	i	1	1
0	0	0	0	1	0	1	1

In K_3, the inference is valid, because for each v with $v((p\wedge q)\wedge\neg p) = 1$ [there is none], we have $v((p\vee q)\supset\neg p) = 1$

3.



Shorter:



All branches close, so the inference is valid

4. No, the inference is not valid.
Take, for example, $v(p) = 1$ and $v(q) = 0.5$
Then $v((\underbrace{p \to q}) \to q) \to q) = 0.5 \leq 0.7$

because $v(p \to q) = 1 - (1 - 0.5) = 0.5$
$v((p \to q) \to q) = 1$ (because $0.5 \leq 0.5$)
$v((p \to q) \to q) \to q = 1 - (1 - 0.5) = 0.5$

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5. $\Diamond(p\wedge\Diamond(q\wedge\Diamond p))\vee(p\wedge\Diamond(q\wedge\Diamond\Diamond p)), 0$

$\neg\Diamond\Diamond\Diamond p, 0$

	$\Diamond(p\wedge\Diamond(q\wedge\Diamond p)), 0$	$p\wedge\Diamond(q\wedge\Diamond\Diamond p), 0$	
	$0r1$	$p, 0$	
	$p\wedge\Diamond(q\wedge\Diamond p), 1$	$\Diamond(q\wedge\Diamond\Diamond p), 0$	
	$p, 1$	$0r4$	
	$\Diamond(q\wedge\Diamond p), 1$	$q\wedge\Diamond\Diamond p, 4$	
	$1r2$	$q, 4$	
	$q\wedge\Diamond p, 2$	$\Diamond\Diamond p, 4$	
	$q, 2$	$4r5$	
	$\Diamond p, 2$	$\Diamond p, 5$	
	$2r3$	$5r6$	
	$p, 3$	$p, 6$	
shortcut versions; long versions also fine	$\neg\Diamond\Diamond p, 1$	$\neg\Diamond\Diamond p, 4$	shortcut versions; long version also fine
	$\neg\Diamond p, 2$	$\neg\Diamond p, 5$	
	$\neg p, 3$	$\neg p, 6$	
	×	×	

All branches are closed, so the inference is valid.

6. To check: $[F]([F]p \supset p) \vdash_{K^t_{\tau}} [F]p$

$[F]([F]p \supset p), 0$

$\neg[F]p, 0$

$0r1$

$\neg p, 1$

$[F]p \supset p, 1$

$\neg[F]p, 1$ $\quad$ $p, 1$ ×

$1r2$

$\neg p, 2$

$0r2$ (τ)

$[F]p \supset p, 2$

$\neg[F]p, 2$ $\quad$ $p, 2$ ×

$2r3$

$\neg p, 3$

$1r3, 0r3$ (τ)

$[F]p \supset p, 3$

$\vdots$ $\quad$ $p, 3$ ×

etc

becomes an infinite open complete branch

The inference is not valid. From the open branch, we can read off the countermodel $I = \langle W, R, v \rangle$ with

$W = \{w_i \mid i \in N\}$

$R = \{\langle w_i, w_j \rangle \mid i, j \in N, i < j\}$

$v_{w_i}(p) = 0$ for all $i \geq 1$

$v_{w_0}(p) = 0 / 1 /$ arbitrary

7. b is complete open branch of a $K_{\tau\sigma}$-tableau and $I = \langle W, R, v \rangle$ is the interpretation induced by b

(i.e: $W = \{ w_i : i \text{ is on } b \}$

$R = \{ \langle w_i, w_j \rangle : irj \text{ is on } b \}$

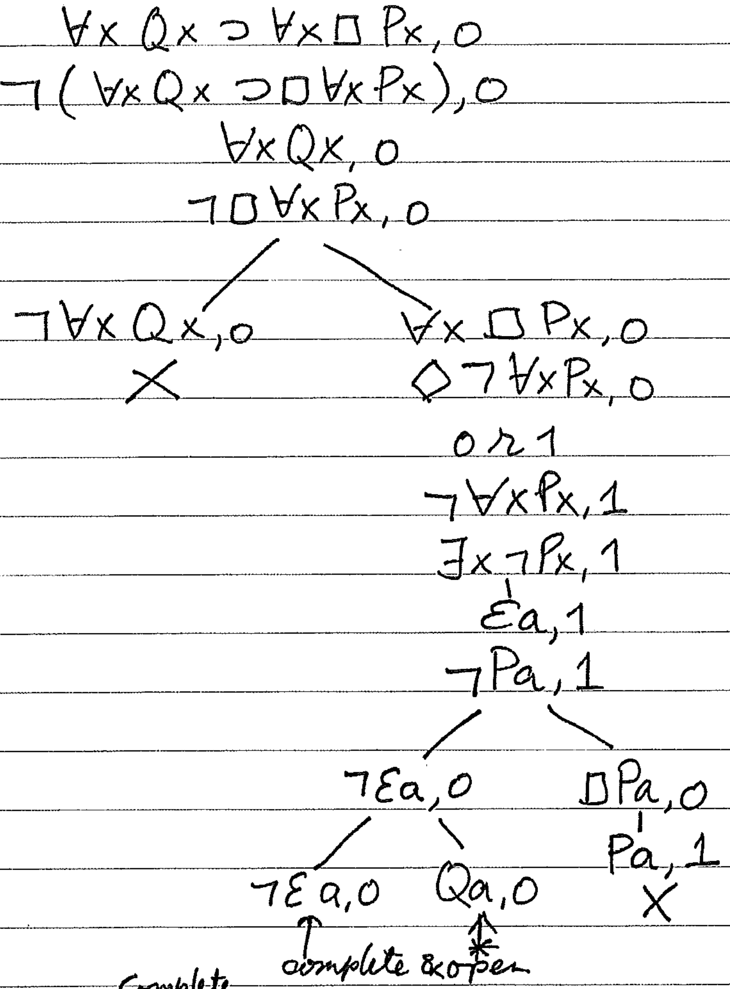
$v_{w_i}(p) = 1$ if p, i is on branch

$v_{w_i}(p) = 0$ if $\neg p, i$ is on branch

Transitive Suppose for arbitrary $w_i, w_j, w_k \in W$, $\langle w_i, w_j \rangle \in R$ and $\langle w_j, w_k \rangle \in R$.
then irj and jrk appear on b.
because b is a complete open branch of a $K_{\tau\sigma}$ tableau, τ has been applied,
So irk appears on b. Therefore $\langle w_i, w_k \rangle \in R$.

Symmetric Suppose for arbitrary $w_i, w_j \in W$, $\langle w_i, w_j \rangle \in R$.
Because irj appears on b, and b is a complete branch of a $K_{\tau\sigma}$ tableau,
σ has been applied. So jri appears on b.
Therefore, $\langle w_j, w_i \rangle \in R$

8. $\forall x Qx \supset \forall x \Box Px \overset{?}{\vdash}_{VK} \forall x Qx \supset \Box \forall x Px$



There are two Complete Open branches, so the inference is not valid. We read off a counterexample from the 'middle' branch with a $*$.

$I = \langle D, W, R, v \rangle$ with

$W = \{w_0, w_1\}$

$D = \{\delta_a\}$ $\quad v(a) = \delta_a$

$v_{w_0}(P) = \emptyset$ $\quad v_{w_0}(\mathcal{E}) = \emptyset$

$v_{w_1}(P) = \emptyset$ $\quad v_{w_1}(\mathcal{E}) = \{\delta_a\}$

$v_{w_0}(Q) = \{\delta_a\}$

$v_{w_1}(Q) = \emptyset$

9. a) (i) The empty sequence is a process, because for all defaults δ_k in it (there are none), 'δ_k is applicable to $In(\Pi[k])$' in an empty way.(*)

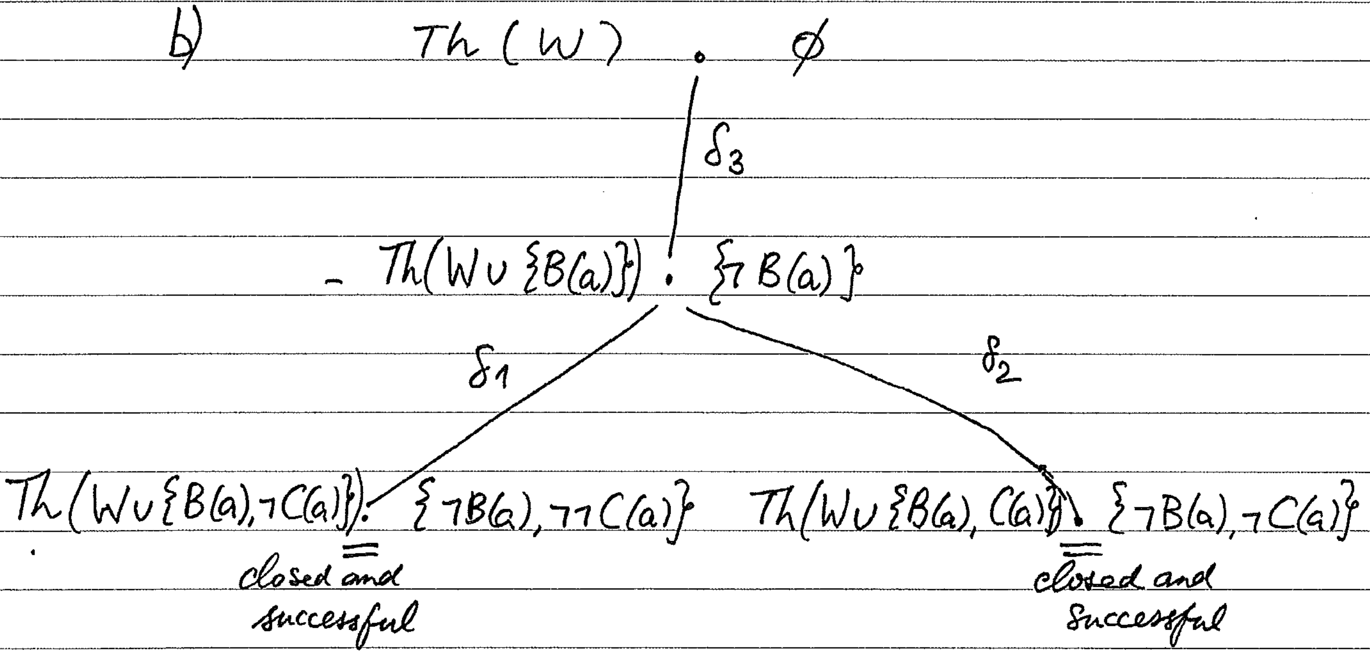
(ii) (δ_2) is not a proces because δ_2 is not applicable to $In() = Th(W)$. This is because $\neg K(a) \notin Th(W)$

(iii) (δ_3, δ_2) is a process because:

- $P(a) \in Th(W)$ and $\neg B(a) \notin Th(W)$, so δ_3 is applicable to $In() = Th(W)$
- $\neg K(a) \in In(\delta_3) = Th(W \cup \{B(a)\})$ and $\neg C(a) \notin In(\delta_3)$ (**)

(iv) • $(\delta_3, \delta_1, \delta_1)$ is not a process, because $\neg\neg C(a) \in In(\delta_3, \delta_2) = Th(W \cup \{B(a), C(a)\})$

b)



(c) The extensions are $In(\delta_3, \delta_1) = Th(W \cup \{B(a), \neg C(a)\}) = Th(\{P(a), B(a), \neg C(a), \neg K(a), \forall x (\neg B(x) \vee \neg K(x))\})$

and $In(\delta_3, \delta_2) = Th(W \cup \{B(a), C(a)\}) = Th(\{P(a), B(a), C(a), \neg K(a), \forall x (\neg B(x) \vee \neg K(x))\})$

(*) ( ) is not closed (because δ_3 can be applied) but is successful, because $Out() = \emptyset$

(**) (δ_3, δ_2) is closed because δ_1 is not applicable, see IV. It is successful because $In(\delta_3, \delta_2) \cap Out(\delta_3, \delta_2) = \emptyset$

Bonus

No, the statement does not hold.

Take $A : p \wedge \neg p$

$B : q \vee \neg q$

Then $p \wedge \neg p \vdash_{K_3} q \vee \neg q$ and $p \wedge \neg p \vdash_{LP} q \vee \neg q$,

but not $p \wedge \neg p \vdash_{FDE} q \vee \neg q$.

Let's make a tableau.

$$\begin{array}{c} p \wedge \neg p, + \\ q \vee \neg q, - \\ p, + \\ \neg p, + \\ q, - \\ \neg q, - \end{array}$$

- The branch is open for FDE, because it does not contain $A, +$ and $A, -$ for any formula. Countermodel: $p\rho 1$, $p\rho 0$, nothing obtains about q
- The branch is closed for K_3, because it contains both $p, +$ and $\neg p, +$. So the K_3-tableau is closed, so $p \wedge \neg p \vdash_{K_3} q \vee \neg q$
- The branch is closed for LP, because it contains both $q, -$ and $\neg q, -$. So the LP-tableau is closed, so $p \wedge \neg p \vdash_{LP} q \vee \neg q$